MATHEMATICAL MODELING OF THE UPPER-INDUS GLACIERS

*Khalid Rashid**

ABSTRACT

Acute water-crisis last summer lead to the desperate suggestion that water needs could be met by melting the water resources frozen as glaciers in northern areas of Pakistan, Northern areas are a precious resource, both for the high lands and the low lands and this resource needs to be sustained for the survival of our civilization. This is of fundamental importance in the light of global warming and world-wide glacier recession. This paper attempts to present the characteristics of the Upper Indus Glaciers and suggest ways to model and understand the northern areas, to help in attaining the sustainability of this resource.

INTRODUCTION

In the spring of 2001, Pakistan was seriously considering the option of melting upper-Indus glaciers to meet the acute shortage of water. Since substantial water flow of the Upper Indus River originates from zones of heavy snow and glacierized basins in Karakorum and Himalayan region, it is no surprise that this idea found its way to near-acceptance at the highest level of authorities. These authorities saw in it a hope to provide a quick relief to water-starved areas. Natural systems, like the northern areas of Pakistan are non-linear systems that are in a quasi-steady state. Tampering with these systems would disturb the balance and result in serious and unforeseen disastrous ecological and social consequences. Amazing, however, is the fact that this idea caught the fancy of the authorities at a time when the Glaciers in the Himalayas are melting at a rapid rate, while nations debate cuts in greenhouse gases (Marquand). These 15,000 glaciers constitute the largest body of ice in the world, apart from the two polar ice-caps. Their runoff feeds the Indus and the Ganges rivers, whose waters sustain 500 million people in the Indo-Gangetic plains

The very preposition of this idea to melt glaciers reveals the lack of basic understanding of the Upper-Indus Basin glaciers and their relationship with the Indus-valley river system, the life giver to the Indus Civilization that now forms Pakistan. The northern areas, its high mountains, glaciers and deep valleys, are a very precious resource for both the highlands and the lowlands and this resource we must understand as best we can, so that we are able to utilize it in a sustainable way. To understand and model the northern areas, much basic research should be undertaken. This is no easy task; yet it is absolutely necessary for the survival of Indus Valley Civilization. In this paper, I will present a simple introduction to glaciers, a numerical example of glacial motion and a general overview of the Upper-Indus Basin glaciers and suggest steps to initiate research and modeling of this system.

FORMATION OF GLACIERS

A large mass of ice that is on land, and shows evidence of being in motion or of once having moved, is called a glacier. The movement of glaciers is now invested with a new and practical interest for humans: early warning of global climatic changes may be indicated by advances or retreats of glaciers. Glaciers exist on all continents, except Australia, and at virtually all latitudes from the tropics to the poles. High latitudes and high altitudes have something in common, for they are both cold. Mountain glaciers, such as those that exist at higher elevations in mid-latitudes and tropics, are particularly sensitive indicators of climate-change.

Glaciers do not just freeze, they grow by a gradual transformation of snow into glacier ice. A fresh snowfall is a fluffy mass of loosely packed snowflakes, which are small, delicate ice crystals, grown in the atmosphere. As the snow ages on the ground, for weeks or months, the crystals shrink and become more compact, and the whole mass becomes squeezed together into a more dense form, called granular snow. As new snow falls and buries the older snow, the layers of granular snow are further compacted to form a much denser kind of snow, usually a year or more old, with little pore-space. Further burial and slow cementation –a process by which crystals become bound together in a mosaic of inter-grown ice crystals-finally produce solid glacial ice. The whole process may take from a few to twenty years. The snow is usually many meters deep, by the time the lower layers are converted to ice.

* Professor, Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan.

The formation of a glacier is complete when ice has accumulated to a thickness sufficient to make the ice move slowly under its own pressure. When this point is reached, the ice flows downhill, either as a tongue of ice filling a valley or a thick ice-cap that flows out in all directions from the highest central area where most snow accumulates. In mid-latitudes, such as Karakorum region, ice melts and evaporates as it flows to lower elevations. Of the total amount of water on earth (1388 10^{15} m^3) 97.3 % is in oceans and 2.7% in fresh-water reservoirs. Of this 2.7 %, the glaciers account for 77%, i.e. 29 10^{15} m^3, acquifiers 22%, lakes and rivers 0.0053% and 000345% is in the atmosphere. If all the glaciers were to melt, the sea level would rise by about 70 meters worldwide. Changes in glaciers seem to be quite sensitive to global climate-changes. Of the numerous physical systems on Earth, glaciers are one of most responsive to climate-change. This is reason enough to study and understand the changes seen and expected in the Karakorum/Himalaya glaciers.

THE BUDGET OF A GLACIER

During winter, the typical glacier grows slightly as snow falls everywhere on the ice surface. In summer, the glacier shrinks, mainly as the snow on the surface of the lowermost reaches melts and evaporates to uncover solid ice, while the upper reaches stay snow-covered. The annual growth budget of a glacier is the amount of solid water added by snow, the **accumulation**, minus the amount lost, called **ablation**. The difference between accumulation and ablation is a measure of either growth or shrinkage of a glacier. Glacier budgets fluctuate from year to year, and many show long-term trends of growth or shrinkage, in response to climate variations over periods of many decades. In temperate climates, ablation occurs mainly by melting under the Sun's rays and less by evaporation. Warm air may blow over lower regions and speed the melting further; the air becomes chilled in the process. If the air is humid, it may precipitate rain, causing even more ablation. The melt waters from Upper Indus Basin glaciers and high altitudes in northern areas are the main source of water in the Indus and its western tributaries and are therefore a question of survival for Pakistan.

HOW ICE MOVES

Once the ice on a slope builds to a great enough thickness, it moves throughout its bulk by internal sliding or flowing movement, as well as its base. The internal flow throughout the ice accounts for much of its motion. Under the stress of its weight, the individual ice crystals slip tiny distances of about 10^{-7} of a millimeter in short time intervals. The sum total of all these movements of enormous number of ice crystals over longer time periods, amounts to larger movements of the whole mass. This movement is similar to the movements shown by some metals, which slowly creep when subjected to a strong stress. Other processes that give rise to movement are also at work. Ice crystals tend to melt and recrystalize a microspcopic amount farther downslope, and other crystal disortions result in movement.

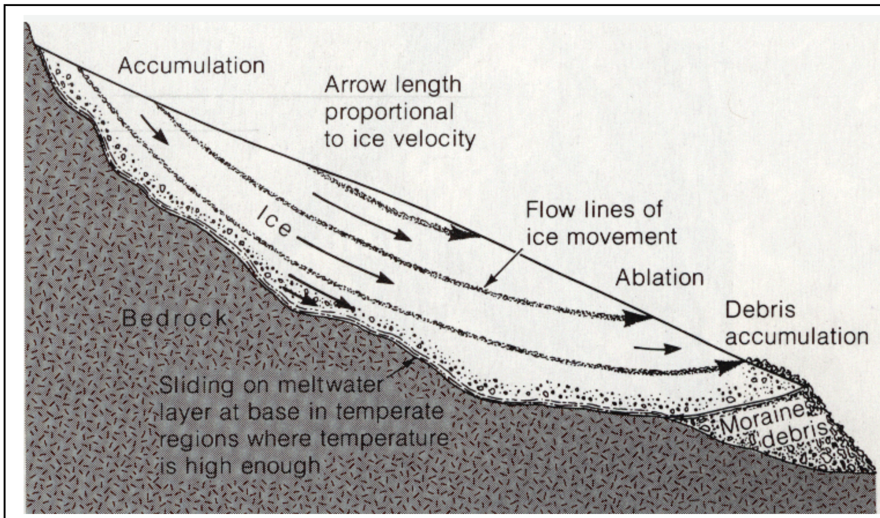

Figure-1: How ice flows in a typical valley glacier. The rate of movement decreases towards the base. If the temperature at the base is sufficiently high that the ice pressure will causing melting, the entire thickness of the glacier will start sliding along the liquid layer next to the ground (Press and Siever).

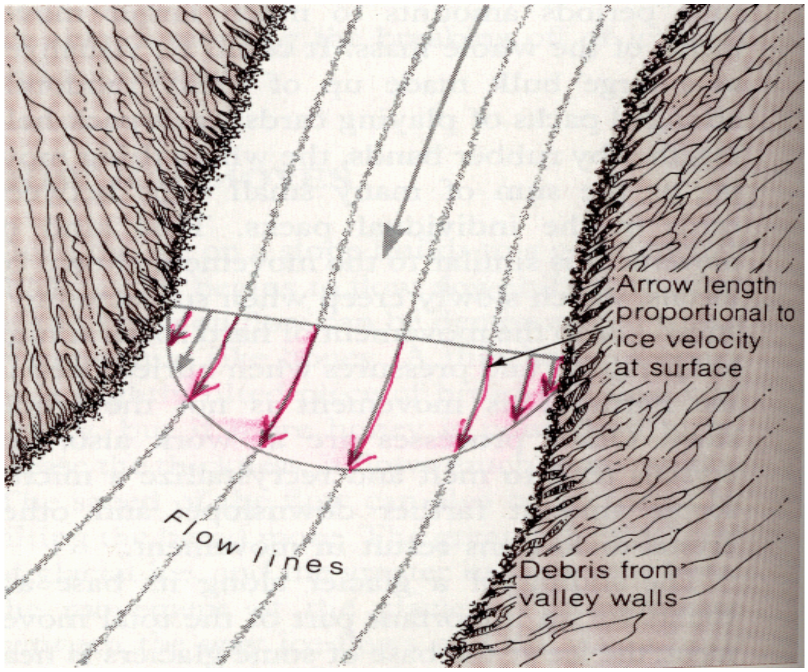

Figure-2: A Valley Glacier Moves More Rapidly At The Centre And With a Greatly Reduced Velocity Along The Valley Walls. *From (Press and Siever)*

The sliding of a glacier along its base accounts for an important part of the total movement. The ice at the base of some glaciers is near the melting temperature, and much of the movement takes place there. Some of the sliding is caused by the melting and refreezing of the ice at the base. The centre of the tongue of ice moves faster that the edges where friction of the ice against the rock walls hinders the flow. Typically, a rapid movement is about 75 meters in one year.

Sudden movements of glaciers, called **surges** occur after long periods of slow movement. Surge may last for one to three years and travel at more than 6 kilometers in one year. Surges may be caused by increase in melting of the base of the glacier, allowing it to slide rapidly, or intermittent releases of ice that piles up in the middle parts of the glaciers while the lower parts are melting. Surges in the Upper Indus Glaciers are relatively frequent and catastrophic. This is not surprising, in view of the large differential in climate along the length of the glacier, because at places the elevation changes are very rapid. The Kutiah Glacier in Karakorum holds the record for the fastest glacial surge. In 1953, it raced more than 12 kilometers in three months, averaging about 113 meters per day.

AN EXAMPLE FROM THE THEORY OF GLACIAL MOTION

A non-linear differential equation in two independent variables furnishes a mathematical description of the conditions in a glacier, particularly in the glacier tongue, or ablator, and appears in the following form (Finsterwalder)

$$[(n+1)\kappa u^{n}-a]\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=-a \qquad (1)$$

It pertains to a central longitudinal section of a glacier moving down a slightly inclined, straight bed.

The variables *u* and *x* refer to oblique axes in the plane of the section: u(x,t) is the vertical depth of the ice at time *t* at a distance *x* along the bed. (Figure-3). The velocity distribution is assumed to be of the form $v = \kappa u^{n}$, where κ depends on the slope of the bed and the exponent n is a constant lying between $\frac{1}{4}$ and $\frac{1}{2}$. The remaining symbol is an ablation constant; it represents the annual melting on horizontal surfaces. This particular

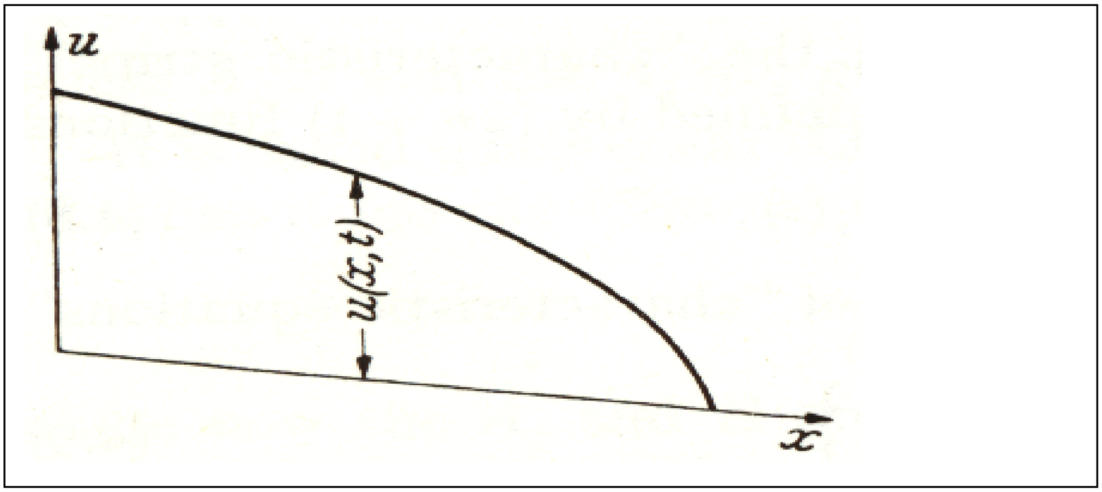

Figure-3: Longitudinal Section of a Glacier

example here concerns a receding glacier on a very flat bed.

Let the shape of the longitudinal section of the glacier at time t = 0 be given in dimensionless variables by

$$u = 2\frac{4-x}{5-x} \quad \text{for} \quad 0 \le x \le 4 \qquad (2)$$

and for numerical values of the parameters take $n = \frac{1}{3}$, $\kappa = 0.075$, $a = \frac{1}{2}$. Then Equation (1) becomes

$$[0.1\sqrt[3]{u} - 0.5]\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = -0.5 \qquad (3)$$

The characteristic equations for equation (1) read

$$\frac{du}{dx} = -a, \quad \frac{dx}{ds} = (n+1)\kappa u^n - a, \quad \frac{dt}{ds} = 1. \qquad (4)$$

Their general solution is a two-parameter family of curves in the (u,x,t) space; in this case the curves are plane and are given by:

$$x + \xi = -\frac{\kappa}{a}u^n + u = u - 0.15u^{\frac{4}{3}}$$
$$u + \eta = -at \qquad = -0.5t$$

where ξ and η are the parameters. From this two-parameter family, we select the one-parameter family of curves which passes through the points of the initial curve given by equation (1); the surface formed by the totality of these curves constitutes the solution of the problem. We select a set of points on the initial curve and calculate the corresponding characteristic parameters; for example the point *t = 0, x=3, u=1* yields $\xi = -2.15$, $\eta = 1$. Then the projections of these characteristic curves may be drawn in a (u,x) plane and graduated at (say equal) intervals in t; curves joining the points with the same values of t give the shape of the longitudinal section at various values of t. (Figure-4).

For a rapid quantitative survey we can solve the equation (1) by finite difference method and avoid the laborious integration of the characteristic equations. The details of the numerical solution, the algorithm and the computer program will be the subject of a specialised presentation elsewhere. Here we present the results geometrically in Figure-4.

THE UPPER INDUS BASIN GLACIERS

The UIB(Upper Indus Basin) in the Karakorum and Himalaya mountains constitutes the heaviest snow and glacial cover on mainland. (See Figure-6).

This region has the largest concentration of peaks higher than 6000 m; the glaciarised

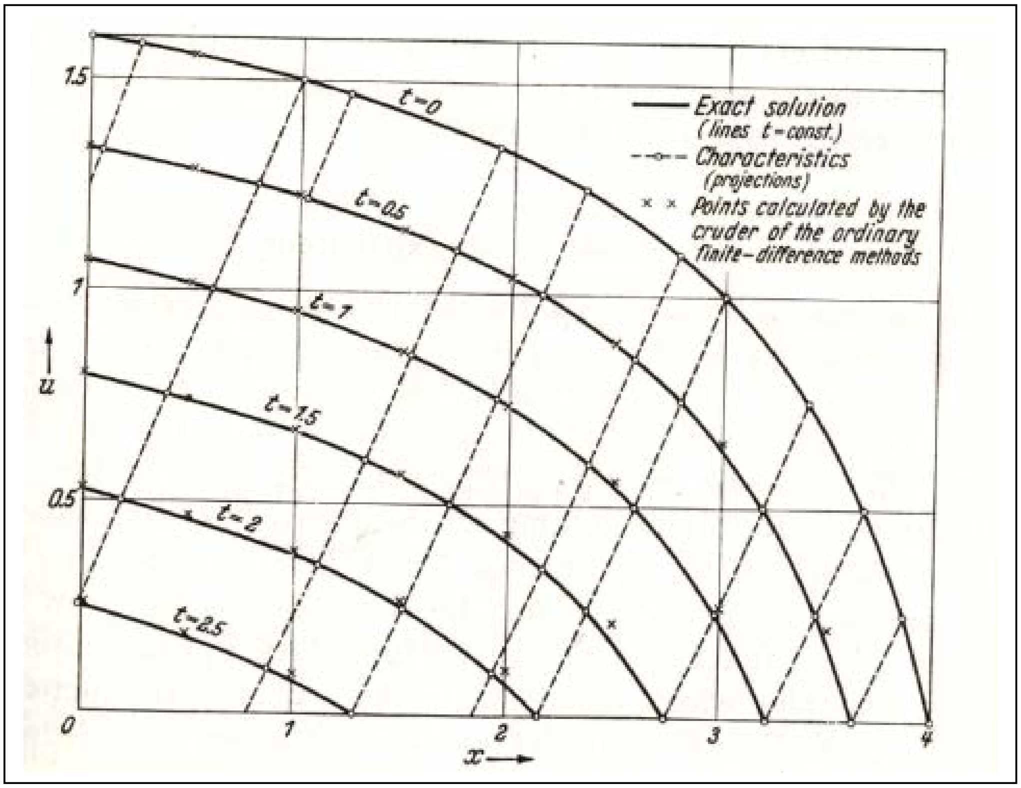

Figure-4: Profile of a Glacier at Various Times as an Example of The Integration of A First Order Partial Differential Equation

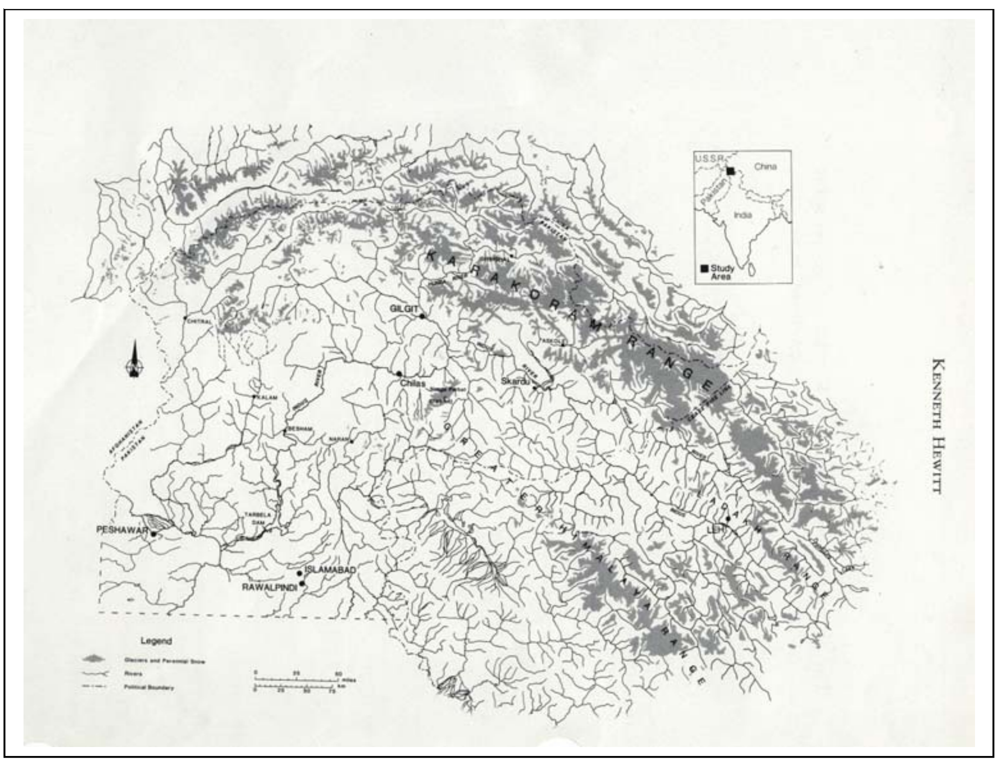

Figure-5: The Karakoram Range Showing The Extent of Perennial Snow and Ice, and The Major River Systems. *(From Hewitt 1989a)***.**

basin area is about 15150 km square, with total snow and ice-cover of 22000 km square. The bulk of the glacier landscape lies between 3000 to 6000 m elevation. There are many glaciers of large size, exceeding 25 kilometer in length.

One special characteristic of the UIB glaciers is the large elevation range from the terminus to the highest parts of the watershed. For some glaciers, this range exceeds 4500 meters. These glaciers are subject to three different weather-systems, namely extremely cold, temperate and semi arid. These features make the Karakorum glaciers different form glaciers in North America, Japan, New Zealand, and even Pamir and Himalayas. For this reason, it is instructive to pay attention to Figure 6 that shows the variation in January and July average temperatures with height for 30^{O}N(Neilburger 1973).

year. So the conditions above an altitude of 4500 meter are conducive to glacier-growth, particularly in winter, if there is enough precipitation. Melting takes place at the middle and lower zones, i.e. the ablations zones, of the glaciers between 4500 to 3000 meters above sea level, mainly in the months of June, July and August. In the accumulation zone, with an altitude greater than 5000 meters, the temperatures do not rise above the freezing point. So the growth of the glaciers take place in this zone. From here, there is a flux of ice to the lower altitudes ablations zones where most of the melting takes place in the summer months. Climate gradients are difficult to understand. Karakorum has the largest range of altitude-variations and atmospheric changes with altitude and precipitation, as a function of altitude, needs to be understood. In Karakoram the gradients dominate the glacier hydrology.

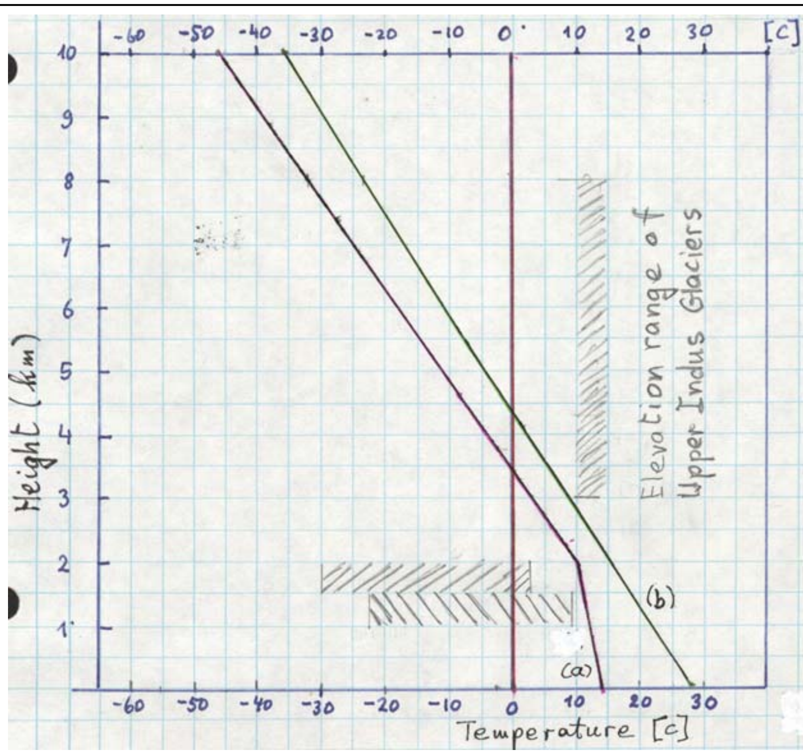

Figure-6: Temperature Variation With Height. (a) January Average. (b) July Average.[////] is the Temperature Range from 3000 to 8000 Meters Above Sea Level In July. [\\\\\] Is The Range In January.

We note that the January average temperature is below freezing at about 3000 meter above see level and in the summer month of July this is the case at about 4500 meter above see level. Large parts of UIB glaciers lie in a temperature range below freezing through the

One of the most important elements of UIB Glaciers that we need to understand is the measurement and quantification of melt water that can depend on various external factors, such as rain and temperature variations and internal factors, such as ice pressures at the base of the glaciers, debris cover, and surges. Snow-melt and melt-waters from this glacier-zone provide half or more of the flows of the main Indus(Hewitt 2001). In Figure-7 we show the hydrographs of Hunza, Kabul and Indus rivers. The discharge increases very substantially in the months of May to September. These are the months when snow- cover disappears from areas below 4500 meters above sea-level and is the time when rapid melting takes place between 3000 and 4000 meters on the middle and lower zones of the glaciers. A detailed study of melt-water for the Biafo glacier basin has been carried out by Hewitt (1989b). Being an important source of water for the Indus River, the understanding of these investigations is a necessary exercise at national level that cannot be over-emphasized.

Biafo glacier is an important source of water for the Indus river. For the Hunza river, the contribution of the melt-water is, on a relative scale, more than that for the Indus and Kabul rivers, as we can infer from the locations of the discharge maximum. For Hunza River the maximum lies in early June whereas the maximum for Indus and Kabul rivers is in June and July. In June and July, monsoon rains also contribute to the discharge more than in June, which is a monsoon-weak month.

Studies with extensive measurements of ice-loss in the ablation area of Miar glacier in the Karakorum mountains have been made as a part of a joint Canada-Pakistan Ice-Hydrology Project, whose aim was to estimate ice-loss in ablation-areas of glaciers, to predict the stream flows (Young, Schmok 1989). I am not aware if this Project is still ongoing. Needless to say, such studies for the Korakorum glaciers must be undertaken as a national or an international effort, to achieve an understanding of UIB glaciers and stream-flows.

A very necessary national task would be to estimate the total ice-flux from high- altitude accumulation zones to lower altitude ablation zone for the UIB glaciers, because this number is an important measure required in the estimation of overall stream flows. There is an urgent need for extensive field-observations and measurements. These can be very difficult to perform, in higher latitudes, because of the difficult terrain and in the ablation-zones because of the large number of crevices. A fruitful approach would be to train

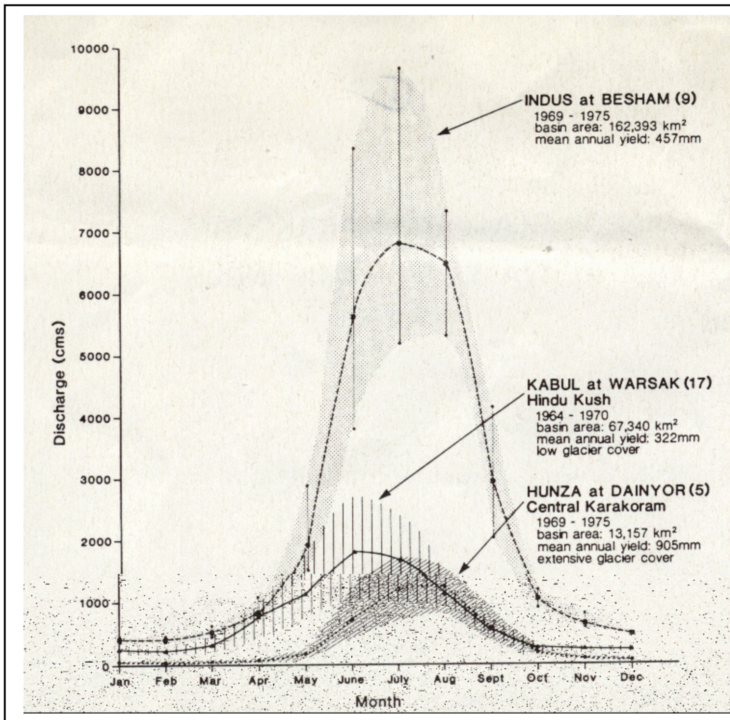

Figure-7: Hydrographs of Indus, Kabul and Hunza Rivers *(from Hewitt 1989b).*

local people of these areas to perform these observations and measurements.

ARTIFICIAL MELTING OF GLACIERS TO MEET WATER SHORTAGE

In the spring of this year, Pakistan was facing nearly 30 MAF(Million Acre Foot) of water-shortage for irrigation, about 50 % of its total needs. Farmers, people and, of course, the Government were deeply concerned. It was in this backdrop that the daily paper THE NEWS of 12-03-2001 carried the following story by G N Mughal. Quote. *Pakistan may use laser technology to melt glaciers. HYDERABAD. Pakistan is seriously considering the option of using laser technology to melt some of its northern areas glaciers and snow lying underneath.* End quote.

My first reaction to this story was to take it as the usual high-level rhetoric of our powerful authorities to calm the thirsty and desperate farmers. Later, when I heard from reliable sources of the seriousness with which this option was under consideration, I was truly shocked. I have never heard of any attempts to melt artificially the snow and ice cover. Before looking at technological details of such an option, let us work out the energy balance of such an experiment. The energy required to melt 1 cm^3 of ice is 333 Joules. So to melt 30 MAF(about 4.2 km^3) of ice 1.2×10^{19} Joules of energy are required. Any good CO_2 laser can have a power of 1000 kW and an efficiency of 20%. Assuming that we have excellent optics to illuminate 100 cm^2 of ice area and that we have the technological capability of operating the lasers, and we want to melt 30 MAF of ice in one month, the electric power required comes out to be 23,000 one-thousand megawatt power plants! If we suppose that we are a super advanced civilisation that could provide this much power, the cost at 5 rupees per Kwh would come out to be 8.28 million billion rupees or 133 trillion US dollars. Who could foot this bill?

Another idea that made the headlines quoting Chief Engineering Advisor, Government of Pakistan, for artificial melting of snow was to spray glaciers with charcoal dust. Dusting of the ice surface with soot has been shown to accelerate greatly the spring-melt over small experimental areas, particularly in polar latitudes, by lowering the albedo substantially (Arnold, 1961). Taking the solar flux at the surface of the earth to be 700 watts/meter square, the albedo is lowered to zero (i.e. total absorption of solar energy) and no cloud cover at all (something quite seldom), then to obtain 30 MAF of water one would have to spray 15,000 km^2 of snow and ice cover. So artificially melting snow and ice cover is a science fiction story!

CONCLUSION AND RECOMMENDATIONS

The glaciers of northern areas are an extremely precious resource, for the highlands and the lowlands, and this resource we must understand so that we can preserve it and utilize it in a sustainable way. At the moment, the area is like a black box, the behavior of which we can only unravel by undertaking basic research of these mountains.

There should be ongoing monitoring of rates of snow-melt and snow-coverage in the Upper Indus Basin Glaciers and of the high altitude snow peaks. So, we need to set up monitoring stations. This is no easy task but, without monitoring, there is no chance of ever understanding this region. An essential ingredient of monitoring is trained manpower. There are very few persons trained in snow and ice hydrology. So we need to train manpower. One way to do this would be to support financially and technically, with infrastructure, M-Phil and PhD thesis related to these areas. It would be very worthwhile to hold an international conference on Karakorum Mountains in the year 2002, not only to hear from the experts about the past, present and future of this area, but to commemorate the year 2002 as the year of the mountains. Once some manpower becomes available, a national Mountain Research Institute should be established. At present, there are no stations directly measuring the high-altitude climatic conditions, i. e. precipitation and temperatures. Finally, to model water run-off from the Upper Indus Basin Glaciers, an integrated approach would be required incorporating Data collected by satellites, ground measurements and global climatic trends.

Majority of the Pakistanis and the Government decision-makers are unaware of the lack of understanding of glaciology and hydrology of these glacier regions. In the absence of monitoring-system, no forecast is possible. It appears that the Himalayan glaciers are diminishing, but the Karakorum glaciers are expanding (Hewitt 2001). Whether this is good news or bad news, we can only determine by working out the end effect on the amount of melt-waters and their reaching the alternate users.

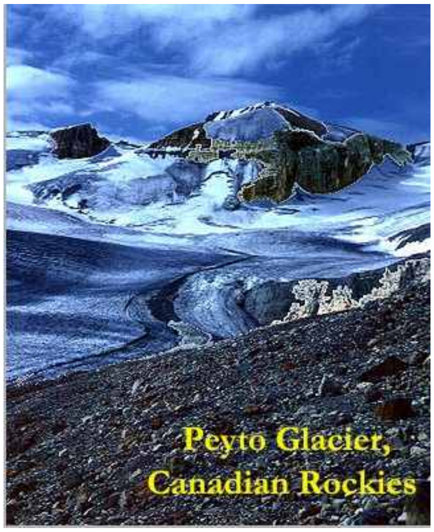

Figure - 8: Peyto Glacier, Canadian Rockies

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